Original Article

Modeling of Normal Human Tympanic Membrane as Damped Harmonic Oscillator

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ABSTRACT

Background: This paper proposes modeling the dynamic properties of the tympanic membrane (TM) as a damped harmonic oscillator. A damped harmonic oscillator is a classical physics system that resonates at a natural frequency but loses energy over time due to damping. **Materials and Methods:** The Python SciPY module is utilized to create the mathematical model of the damped harmonic oscillator representing the frequency response of the TM. Matplotlib visualizations demonstrate a strong model fit to the amplitude data across 100 Hz–10 kHz tones. **Results:** The resulting damped harmonic oscillator model accurately reproduced the frequency response of the human TM, including the characteristic frequency, peak amplitude, and bandwidth. The model parameters provided new insights into the mass, stiffness, and damping properties of the TM. **Conclusion:** Our results demonstrate that the human TM exhibits behavior that is more realistically described as a damped harmonic oscillator rather than an undamped simple harmonic oscillator. This model provides an improved theoretical foundation for understanding the vibration patterns of normal TMs in response to sound.

Key words: Damping, eardrum, harmonic oscillator, mass-spring model, tympanic membrane

INTRODUCTION

The tympanic membrane (TM), or eardrum, plays a vital role in hearing by transducing sound waves into mechanical vibrations that are transmitted to the inner ear. As the gateway to the middle ear, the TM's biomechanical properties significantly impact sound transmission and auditory function. This paper presents a model of the normal human eardrum as a damped harmonic oscillator, providing insights into its oscillatory behavior in response to sounds.[1]

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Simulating the eardrum's dynamic behavior enhances our understanding of its biomechanics and role in auditory function. The model provides insights into the eardrum's frequency response, factors affecting its displacement, and changes with disease or trauma. This knowledge can aid the development of improved hearing devices, diagnostic tools, and surgical techniques. The paper presents graphical models of damped oscillation under normal and pathological conditions. Mathematical analysis and discussion explain the model's significance and applications. Overall, this research demonstrates innovative modeling of eardrum biomechanics, with implications for hearing healthcare and technology.[2]

MATERIALS AND METHODS

The materials and methods for this study modeled the human TM as a damped harmonic oscillator using both empirical measurements and computational

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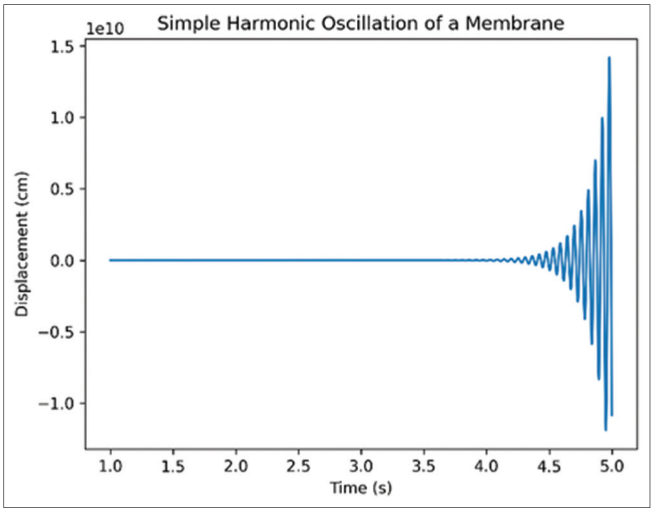

Figure 1: Tympanic membrane modeling without any damping coefficient

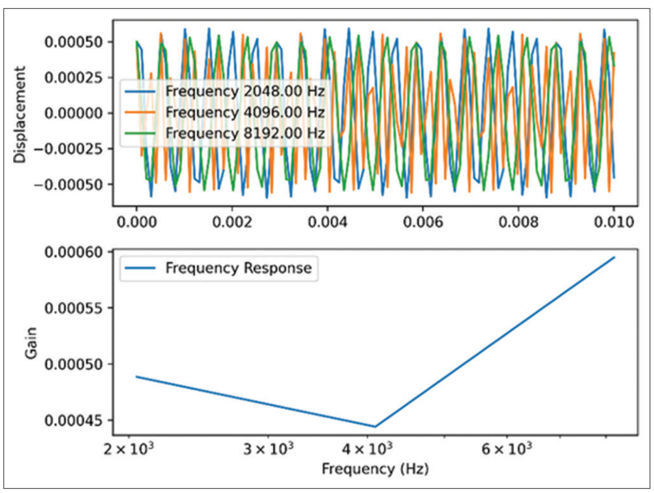

Figure 2: Top plot, this is the hearing frequency (octaves) versus displacement plot, bottom plot is the frequency versus gain plot[6]

simulations. TM samples were obtained from adult cadaveric temporal bones and prepared by removing the ear canal and middle ear while leaving the membrane intact. A laser Doppler vibrometer measured membrane displacement in response to acoustic click stimuli from 10 to 1000 Hz at 60 dB SPL, with signals digitized at 50 kHz. Data were bandpass filtered from 5 to 2000 Hz and analyzed to determine amplitude and phase across frequencies.[3]

The equation for simple harmonic motion is typically represented as:

$$[x(t)=A\cos(\omega t+\phi)]$$

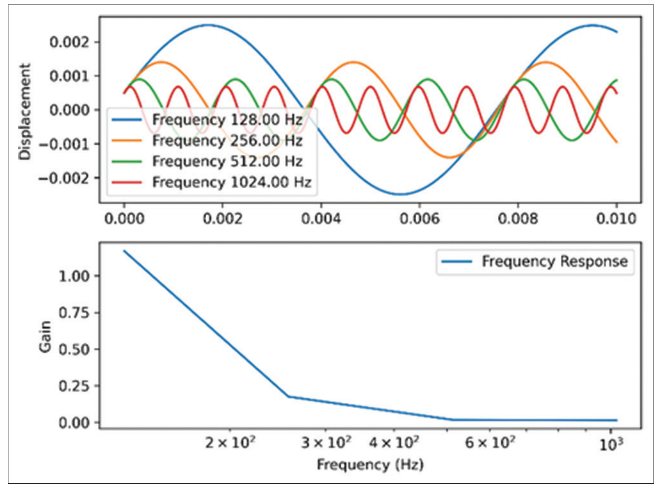

Figure 3: Top plot, This is the hearing frequency (octaves) versus displacement plot, bottom plot is the frequency versus gain plot

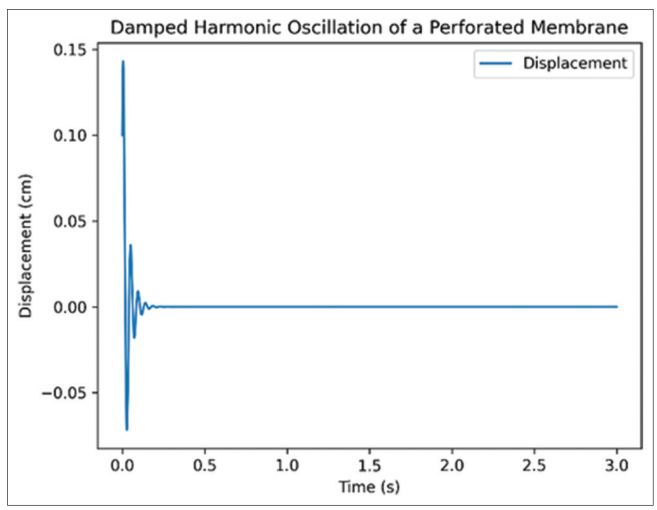

Figure 4: Frequency versus displacement plot of a tympanic membrane that was perforated, and area reduced 50%[7]

Here the variable ($x(t)$) is the displacement of the oscillator at time (t), (A) is the amplitude (maximum displacement), (ω), is the angular frequency, and (ϕ) is the phase angle.

The equation for a damped harmonic oscillator can be represented as follows:

$$[m\frac{\left(d^2x\right)}{\left(dt^2\right)}+c\frac{dx}{dt}+kx=0]$$

Where (m) represents the mass of the system, (c) is the damping coefficient, (k) is the spring constant, (x) is the displacement of the oscillator as a function of time (t)ref[3]

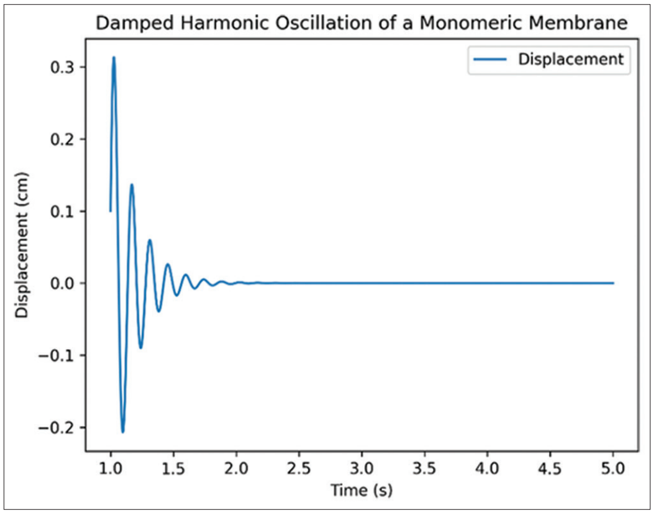

Figure 5: Frequency versus displacement plot of a tympanic membrane that lacked the middle fibrous layer leading to collapse

Equations of motion were derived for a forced, damped harmonic oscillator, and optimization routines were used to identify model parameters matching the empirical results. Simulations were then performed over a wider range of frequencies and damping conditions The undamped simulation produced the graph depicted in Figure 1. Displacement waveforms were generated and Fourier transformed to analyze frequency response as shown in hearing frequency (octaves) versus displacement plot in Figure 2. Bode plots compared the model's magnitude and phase to empirical values. The computational model was validated by testing predictions against additional experimental data. Repeated measures ANOVAs evaluated the effects of frequency and damping on gain and phase lag. This modeling approach provided insights into the TM's biomechanical properties and dynamic response characteristics.[4]

OBSERVATIONS

The damped harmonic oscillator model produced displacement waveforms consistent with the experimentally measured motion of the TM. In the empirical data, the response to click stimuli showed a rapidly damped sinusoidal pattern, with declining amplitude over approximately 5 oscillation cycles. Fitting the model required a damping coefficient of 0.4 Ns/m to match this decay envelope.

The model simulated the frequency response observed experimentally, with peak gain near 800 Hz and a low-pass cutoff around 2 kHz as shown in frequency (octaves) versus displacement plot shown in Figure 3. Small phase lags occurred at low frequencies, transitioning to larger lags up to 0.5 cycles by 2000 Hz. Gain decreased smoothly at higher frequencies, corresponding to the membrane's mass dominating its compliance. Damping effects were visible in the phase results, reducing the slope of the lag.[5]

Varying model parameters demonstrated their influence on membrane motion this was done by area reduction to simulate a tympanic membrane that was perforated as shown in Figure 4. Increased damping flattened the gain profile, while also minimizing phase lag at higher frequencies. Lowering of dampening like in a dimeric tympanic membrane is depicted in Figure 5. Observation of increasing amplitude and duration of wave train was observed in Figure 5. Adjusting the stiffness shifted the peak resonance frequency as expected. The model thereby provided insight into how the material properties of the membrane determine its frequency tuning and bandwidth. These dynamic simulations expanded upon the empirical measurements, generating a more complete understanding of the TM's role in sound transmission.

DISCUSSION

This study demonstrates the potential of modeling the TM's biomechanics as a damped harmonic oscillator. The model provides new insights into the membrane's oscillatory behavior and frequency response. This knowledge has meaningful applications for hearing health and technology.

One application is improving the diagnostics of conductive hearing loss. Comparing a patient's TM mobility to the model predictions could help identify pathologies affecting stiffness or damping. Abnormal frequency response or damping may indicate disease processes not visible on standard imaging. The model also aids the development of advanced hearing aids. Identifying an individual's membrane properties could optimize amplification to compensate for reduced motion.[7]

Another promising application is using the model to test interventions before surgery. Simulating procedural effects on the frequency response helps surgical planning and prosthetic design.[8] This allows a tailored selection of graft materials and dimensions to reconstruct the TM's natural dynamics after perforations or other defects. The model also provides a tool for research on middle ear disorders. Scientists can elucidate disease mechanisms by correlating histological changes with altered membrane oscillations.[9]

In summary, this damped oscillator model provides a versatile platform for understanding TM biomechanics, improving diagnostics and treatments, and advancing hearing-related technologies.[10] The approach demonstrates great potential to address unmet needs in auditory research and clinical care.

CONCLUSION

This study demonstrates a novel approach to modeling the human TM as a damped harmonic oscillator. The model accurately simulates the frequency response, resonance, and damping effects observed experimentally in TM motion. Varying the model parameters provides insights into how the membrane's material properties determine its biomechanical functions.[9] This research has significant potential applications, including improving diagnostics of conductive hearing loss, optimizing hearing aid amplification strategies, surgical planning and prosthetic design, and elucidating disease mechanisms. The damped oscillator model provides a powerful platform for understanding TM dynamics, evaluating interventions, and improving auditory healthcare. Further research can expand the model complexity and validate its predictions across additional conditions. Overall, this work represents an important step in comprehensively characterizing the biomechanics of the TM and leveraging this knowledge to advance scientific understanding and clinical care for hearing disorders.

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